Novel Microhertz Comparison Measurement of Remote Frequency Standards

Fast Micro-Hertz Measurement of Frequency by Phase

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***Abstract*—Measuring frequency by phase is demonstrated practically to be greater than a hundred times faster than direct frequency measurement. Even on remote off-air radio frequency standards affected by radio propagation variations, microhertz accuracy can be achieved in hours rather than weeks. And this at low cost by using readily available SDR techniques. The novelty is the use of a large (21 bit) high precision (24 bit) FFT on a signal translated to IFs at about 70 to100 Hz and with FFT window lengths of a few minutes. These first results indicate that this novel technique could in the future compute Allan Variances perhaps a hundred times faster than the longest stated 'observation-time'. This frequency-by-phase fast measurement technique is also compared with a frequency-bin-interpolation technique.**

Keywords*—fast frequency-by-phase measurement; SDR techniques; frequency-bin-interpolation technique; fast Allan Variance; radio frequency standards

I. Introduction

The initial aim of this work was to be able to assess the limiting accuracy over a typical day of radio frequency standards as measured after propagation to long distances. It follows on from and substantially improves on the author's previous work on measurement of the propagation of both long-wave and short-wave frequency standards [1-7]. The new technique is here demonstrated on two long wave frequency standards. So far, several weeks of continuous 24-hour measurement plots have been obtained in Lingfield, UK simultaneously for the two long wave frequency standards, on 198 kHz in Droitwich, UK at a distance of 200 km, and 162 Hz in Allouis, France at 470 km.

This paper reports the first practical results of improvements in measurement speed that can be obtained by measuring 'frequency-by-phase'. So far about two orders of improvement are demonstrated with more possible depending on overall noise characteristics of the received signal as measured in different bandwidths. The most significant advance has been made by filtering and processing the signal by a large FFT with window lengths of a few minutes.

Being a novel method there is very little if any previously reported work to be found by other authors on the advantages of measuring frequency by phase. Other papers do provide reference to the latest state of the art in high accuracy frequency measurement against which the frequency-by-phase technique may be assessed [9]. Further references also refer to educational resources such as Wikipedia and other essential practical resources to be found on the internet. These may or may not be regarded as scholarly references. Nonetheless they do allow credit to be given where credit should be acknowledged in reality.

In assembling the measurement hardware and software this author discovered that a fast frequency measurement method put forward by him in 2015 [2] had independently been included as a facility in the Spectrum-Lab Audio Spectrum Analyzer software excellently created by Walter Buescher [12]. Some of the results presented here essentially compare these two fast frequency measurement techniques in some practical cases.

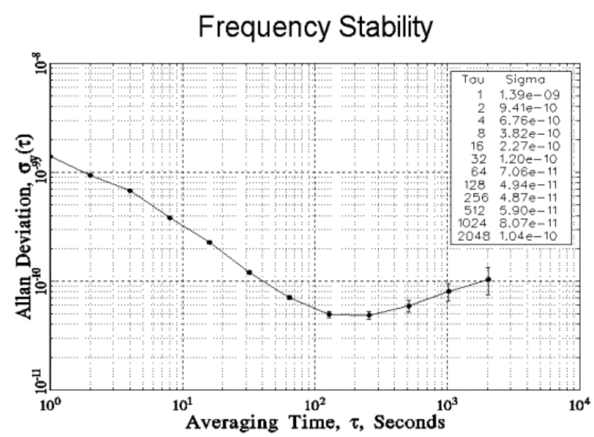

Fig. 1. Example plot of the Allan deviation of a clock. At very short observation time τ, the Allan deviation is high due to noise. At longer τ, it decreases because the noise averages out. At still longer τ, the Allan deviation starts increasing again, suggesting that the clock frequency is gradually drifting due to temperature changes, aging of components, or other such factors. The error bars increase with τ simply because it is time-consuming to get a lot of data points for large τ.
(From https://en.wikipedia.org/wiki/Allan_variance [8]

II. Background and Aims

The Allan Variance is a precise of measure of frequency for a range of two-sample differences at various observation times τ [8]. For a typical source there is an optimum observation time at which the best frequency stability can be measured, as shown in Fig, 1. Essentially the best frequency stability is measured when the drift and phase noise degradations of the Allan variance are about equal. But note that the error bars increase rapidly as the observation time increases. This introduces uncertainty in the drift value and hence in the average frequency estimate. For a given data record of

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measurements, the THEO type of processing algorithm can extend the reliable observation time by about 50% [9].

This paper demonstrates by practical measurements that by use of the concept of measuring frequency-by phase using a large FFT as a filter and integrator about two orders of magnitude further improvement may be obtained even for off-air radio frequency standards. And with low-cost easily available (SDR) hardware and software. Thus, micro-Hertz differences can be measured in ~10,000 seconds or about 2.78 hours rather than 2,00,000 seconds or 23.2 days expected for Allan Variance methods.

III. CONCEPT

Frequency is rate-of-change of phase or 'phase-slope'. Hence it can be said that phase measures 'fractional-frequency' One µHz is 1.3° per hour.

Phase measures frequency (and perhaps also Allan Variance) about 100 times faster than before.

Thus, micro-Hertz differences can be measured in ~10,000 seconds or about 2.78 hours rather than 1,000,000 seconds or 11.6 days required for direct frequency measurement by counting cycles

A stable reference is still needed. But its long-term drift is less important because the required record time is about 100 times shorter. Also reference phase noise is significantly reduced by the FFT filtering used to process the signal spectrum.

IV. TECHNIQUE AND EQUIPMENT

The low-cost equipment used to implement and demonstrate the new technique incudes:

1. SDRplay RSPdx 1kHz to 2GHzSDRs single tuner for single antenna [10].
2. SDRplay RSPduo 1kHz to 2GHzSDRs dual tuner for two separate antennas [10].
3. RSPduo software (free-ware and regularly updated) [10].
4. VB-Audio virtual-audio-cable and audio mixer software (donation-ware) [11].
5. 'Spectrum Lab' audio-spectrum-analyser – free-ware from radio amateur Wolfgang Buescher DL4YHF. Analyses two signals on low- IFs of 70 Hz and 100 Hz [12].

Fig. 2. Screenshot shows SDR controller on two frequencies with spectra shown at bottom right giving low IFs at 70Hz and 100Hz passed via VB-Virtual Audio Cable, controller shown top right to Spectrum Lab software giving 24 hour plots as in Fig. 3 at centre and underneath.

6. Low-cost GPS frequency references designed by Leo Bodmar DG5SAQ available from SDR Kits, <1 in 10^9 stability [13].
7. PCs used are: ACER Aspire R13 or Dell Studio (32-bit).
8. Wellbrook Active H-field Loop Antenna [14].
9. Mini-whip Active E-field Antenna [15].

Carrier signals to be measured are received on either a Wellbrook Active Loop H-field Antenna or a Mini-whip E-field. Or both at the same time using the separate tuners of the SDRplay RSPduo.

Both SDRs used are simultaneously locked to the Leo Bodmar 24MHz GPS-locked reference set for the required SDR reference frequency of 24MHz.

The single tuner RSPdx actually has enough bandwidth (up to 2MHz) to be able to implement two software receivers simultaneously to demodulate the 162 kHz and 198 kHz carriers to 70 Hz and 100 Hz low IFs respectively. But for comparison of the phase differences between the received E and H fields the two-tuner RSPduo is required. (Such measurements are in hand as further work.)

Fig. 1 shows the control panels for the various parts of the measurement system all of which are simultaneously run on a single Laptop computer running Windows 10 as the operating system.

Originally the two low IF signals at 70 Hz and 100 Hz (in digital audio format) were added and conveyed by a freeware VB-Audio Virtual-Audio-Cable (VAC) to the freeware Spectrum-Lab Audio Spectrum analyzer. But now separate VACs from and a 'Voicemeeter' mixer, all from VB-Audio (but as donation-ware), are used so that the two low-IF signals can be balanced when they come from separate tuners [11]. Note that the use of VACs bypasses the SDR sound-card(s) and thus removes any sound card clock drift and drop-out problems, normally encountered in SDRs when precision use is required.

The Spectrum-Lab Audio Spectrum Analyzer created by Wolfgang Buescher [12] is very sophisticated and versatile. It has a 21 bit FFT with 2,097,152 frequency bins that can operate comfortably at sound card clock rates of up to 96 kb/s And this can be easily set to decimate by up to 531441 in useful intervals for micro-Hertz frequency bin width. Normally a decimation range of 16 to 64 is used to give FFT window lengths of about 1 to 4 mHz and about 1000 to 250 second FFT sample rates. The Analyzer has two channels so the 70 and 100Hz signals can be analyzed and plotted separately.

The most important feature of the Spectrum-Lab analyzer is that it can plot absolute phase against a settable NCO (Numerically Controlled Oscillator) in the software. The NCO frequency is set manually to the necessary precision to obtain a phase trace that is as flat as possible for as long as possible. (It takes some skill and practice.) The frequency difference between the measured signal carrier and the internal NCO is given at any one time by the slope of the phase against time. A difference of one µHz is 1.3 degrees per hour.

V. FREQUENCY-BIN-INTERPOLATION TECHNIQE

A second important feature of the Spectrum-Lab analyzer is that it can measure frequencies very fast provided that the closein noise SNR (Signal-to-Noise-Ratio) is good enough. This is now here called 'frequency-bin-interpolation'. It measures the ratio of the two adjacent largest amplitude frequency bins and interpolates between them to give a much more accurate frequency estimate. On low-noise signals frequencies can easily be estimated to better than $1/100^{th}$ of the frequency bin spacing. This method can approach the accuracy and speed of measuring frequency-by-phase but only if the signal (amplitude) noise is low enough.

VI. Measured Results

Several months of measurements have been made with the equipment as described. Figures 3 to 5 have been selected to demonstrate the speed and accuracy that can be obtained from the frequency-by-phase measurement and frequency-bin-interpolation techniques. All three figures are continuous records of about 24 hours of measurements with the most recent times on the right.

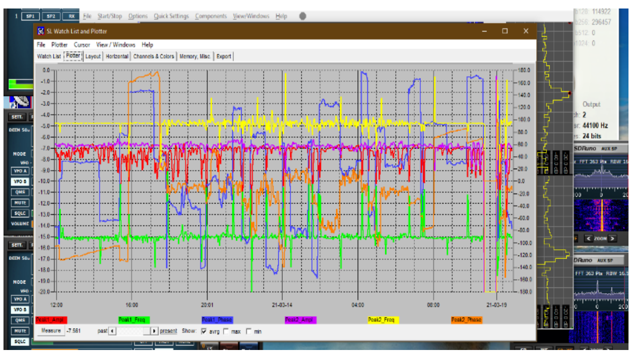

Fig. 3. At centre 24 hour left-to-right Spectrum-Lab plot of two signals, at 198 kHz and 162 kHz superimposed on controllers of Fig. 1. For each is plotted amplitude, frequency and phase (360° p-p scale on right) according to colour scheme on legend, with 198 kHz for the left-hand three. Note the two vertical FFT stepped spectra (at 100 Hz top and 70 Hz at bottom) and just to right of central plot. The FFT frequency bins are 2.86mHz but with a gaussian window smoothing giving a noise bandwidth of 4.15 mHz and an FFT window time of 5.83 min. The yellow and green measurements are obtained faster and to greater accuracy by interpolating between the largest two samples on each FFT. This relies on the signals having low AM noise.

Fig. 3 show the capability of the system in that the two signals can be measured in frequency and phase at the same time and directly compared. The 198 kHz signal is in Droitwich, UK at a distance of 200 km from Lingfield where the measurements were performed. And the 162 kHz signal comes from Allouis, France at a distance of 470 km. There are six plots shown. Red, green and blue are respectively amplitude, frequency and phase for the Droitwich carrier signal at 198 kHz, and violet, yellow and orange are the same three parameters respectively for the Allouis signal on 162 kHz.

The phase jumps and frequency spikes have since been found to be mainly due to occasional overload of the Dell Studio (32 bit) computer and have now been significantly reduced. But nonetheless there are flat sections of phase slopes lasting over an hour or so where the phase slope may be measured with reasonable accuracy to obtain a measure of the frequency stability down to a few microhertz even on these first results.

Also note that the phase noise on the flat sections of the orange trace for 162kHz at 470 km distance is visibly greater than on the blue trace for the 198kHz signal at 200 km.

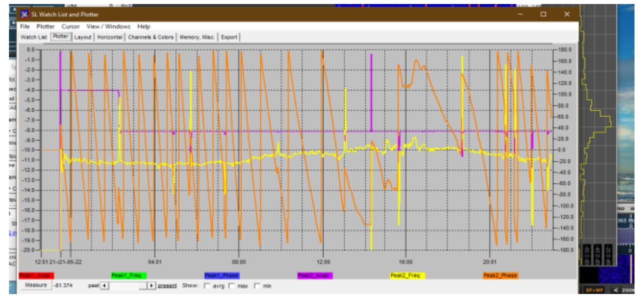

Fig. 4. Frequency (yellow) and phase(orange) differences between GPS phase- locked reference and 10MHz Rubidium source (not double ovened) over 24 hours left to right. The frequency scale vertical span is 100μHz. Diurnal change with Ru source changes by about ±8 μHz for about 10^{0}C p-p temperature change. Phase jumps and frequency spikes are from the GPS phase-locked reference source or from the Dell computer.

Fig. 4 measures a 10MHz signal from a Rubidium locked source with he SDR receiver locked to a GPS locked reference at 24MHz. Frequency is in yellow and phase is in orange. The daily drift of the Rubidium source with temperature, amounting to about ±8 μHz for about 10^{0}C p-p temperature change, can clearly be seen. This is ±8 parts in 10^{13} stability measured over a day. This result is obtained significantly faster than the million seconds (11.6 days) or more normally required for such a measurement.

(A temperature-controlled enclosure is now being constructed for the Rubidium source.)

The phase stability of the GPS clock can be estimated by the variations seen on the orange plot.

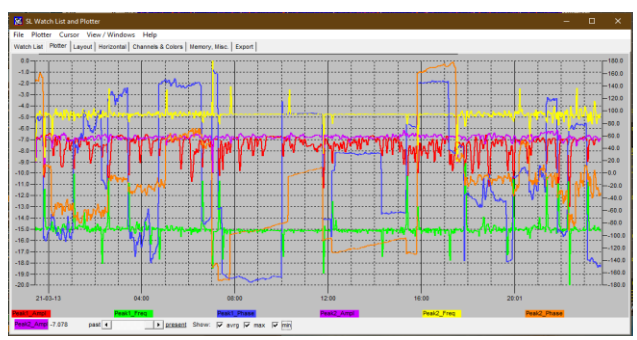

Fig. 5. Midnight to midnight plots of signal-amplitude (scale 0 to 20dB on left), frequency (20mHz p-p scale) and absolute phase (± 180 deg. Scale). Plot colours as on legend for 198 kHz first three on left and 162 kHz last three on right. Note that multi-path phase jumps are larger and much more rapid than expected from current multipath theory.

Fig 5 is a repeat of the measurements made in Fig. 3 after the computer induced glitches have been reduced. The remaining phase jumps are thought may be multipath phase jumps but further work is needed to confirm this. However, the time length between phase jumps is now significantly longer so that the phase slope can be estimated more accurately over longer periods.

VII. Conclusions

The measurements made so far have confirmed that in practice the frequency-by-phase measurement technique is at least two orders faster than direct frequency measurement. At the moment the frequency-bin-interpolation is perhaps a few times slower and it less effective on weak signals with a poor SNR. However, the practical and theoretical limits of both of these methods have not yet been fully analyzed and established. Further work is needed.

These first results indicate that the novel frequency-by-phase technique could in the future compute Allan Variances perhaps a hundred times or more faster than the present limit of the ‘observation-time’.

Both tools will also continue to be used to investigate multipath phase jumps and Doppler frequency shifts particularly for surface wave propagation.

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